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COMMENTS ON "A NOVEL APPROACH TO DETERMINE THE FREQUENCY EQUATIONS OF COMBINED DYNAMICAL SYSTEMS"

M. GÜRGÖZE

Faculty of Mechanical Engineering, İstanbul Technical University 80191 *Gümüşsuyu İstanbul, Turkey. E-mail*: *gurgozem@itu.edu.tr*

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In the context of the derivation of the characteristic equation of a continuous structure represented by an N-degree-of-freedom (d.o.f.) discrete system to which s spring–mass systems are attached, Cha and his co-worker manipulated the corresponding generalized $(N \times N)$ eigenvalue problem so that its characteristic determinant is equivalent to that of a smaller $(s \times s)$ matrix (assumption: $s \ll N$) [1]. In the appendix, they have given the derivation of the corresponding formula for $s = 1$ and stated that the general case of arbitrary s is merely an extension of the given derivation. The aim of this short note is to make a positive comment on the appendix of the above interesting and useful study and to show that the formula for the special case of $s = 1$ can also be obtained in a shorter way directly, though it may not be generalized for any s.

Noting that $\mathbf{K}^d$ and $\mathbf{M}^d$ are diagonal matrices, the problem to be solved is the calculation of the determinant of the matrix $[\mathbf{\Lambda} + \sigma\boldsymbol{\phi}\boldsymbol{\phi}^{\mathrm{T}}]$, where the diagonal matrix $\mathbf{\Lambda}$ is defined as

$$\mathbf{\Lambda} = [\mathbf{K}^d - \omega^2\mathbf{M}^d] = \mathbf{diag}(K_i - \omega^2 M_i) \quad (i = 1, \ldots, N).$$

Using the well-known formula from the matrix theory [2]

$$\det(\mathbf{A} + \alpha\mathbf{b}\mathbf{b}^{\mathrm{T}}) = (\det\mathbf{A})(1 + \alpha\mathbf{b}^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{b})$$

for the determinant of the sum of a regular square matrix $\mathbf{A}$ and a rank-one modification, the following can be written directly:

$$\det(\mathbf{\Lambda} + \sigma\boldsymbol{\phi}\boldsymbol{\phi}^{\mathrm{T}}) = (\det\mathbf{\Lambda})(1 + \sigma\boldsymbol{\phi}^{\mathrm{T}}\mathbf{\Lambda}^{-1}\boldsymbol{\phi}).$$

The second factor yields

$$1 + \sigma\boldsymbol{\phi}^{\mathrm{T}}\mathbf{\Lambda}^{-1}\boldsymbol{\phi} = 1 + \sigma[\phi_1 \cdots \phi_N]\,\mathbf{diag}\left(\frac{1}{K_i - \omega^2 M_i}\right)\begin{bmatrix}\phi_1 \\ \vdots \\ \vdots \\ \phi_N\end{bmatrix} = 1 + \sigma\sum_{i=1}^{N}\frac{\phi_i^2}{K_i - \omega^2 M_i},$$

which is exactly the result given in the appendix of reference [1].

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